A Light Beam Waveguide Using Hyperbolic-Type Gas Lenses

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Abstract—This paper is concerned with the optimum design of a light beam waveguide constructed with a proposed lens-like medium, namely, a hyperbolic-type gas lens which has a hyperbolic temperature distribution on its transverse cross section. In such a medium, the temperature distribution is ideally quadratic in the transverse directions, and so the mode conversion due to the higher order variation of the dielectric constant can be minimized. Moreover, this guide has the following merits; the design procedure is simple, the mode matching at the input part is easy, and it is possible to construct a waveguide using ordinary air as the lens medium. And this consideration can be easily extended to the guide with curved configuration. It was shown that the experimental convergency of this gas lens was in agreement with the theoretical one.

I. Introduction

THERE ARE many reports on a light beam waveguide, and as for the focusing element, a sequence of lenses [1], or reflectors [2], or gas lenses [3]–[5], are considered.

As an example of a beam waveguide using gas lenses, Tien et al., proposed a helical quadrupole lens-like medium for a beam of Gaussian field distribution [5].

This paper is concerned with the analysis and experimental study of a light beam waveguide constructed with a proposed lens-like medium, namely, a hyperbolic-type gas lens (HGL) which has a hyperbolic temperature distribution on its transverse cross section [6]. The transmitted mode is assumed to be a beam of Gaussian field distribution. In such a medium, the temperature distribution becomes exactly quadratic in the transverse directions and the distribution of the dielectric constant becomes very close to quadratic, and so the mode conversion due to the higher order variation of the dielectric constant is very small [7]. And it is possible to have the optimum design of a beam waveguide constructed with this type of gas lenses. At the optimum condition, the maximum beam spot of the transmitted mode in the guide becomes minimum which, in turn, minimizes the diffraction loss.

II. A Beam Waveguide Constructed with Hyperbolic-Type Gas Lenses

A. Hyperbolic-Type Gas Lens

The transverse cross section of a proposed hyperbolic-type gas lens is, in principle, shown in Fig. 1. Two pairs of hyperbolic-type heat conductive pipes are heated or cooled by $+\Delta T$ or $-\Delta T$ °C, respectively, compared with the temperature T_0 on the center axis, and it is assumed that there is no convection. The separation of each pair of pipes is $2a$. Then the temperature distribution in the medium is given by $T=T_0+\Delta T(x^2-y^2)/a^2$ using Laplace's equation.

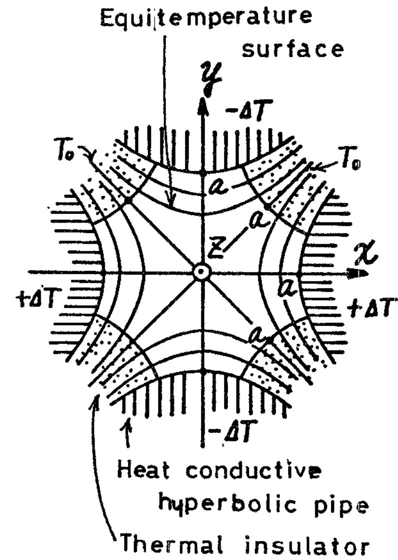

Fig. 1. Cross-sectional view, in principle, of hyperbolic-type gas lens.

In this case, the distribution of the dielectric constant in the medium obeys Clausius-Mossotti's relation and is calculated as follows, under the condition of $T_0 \gg \Delta T(x^2-y^2)/a^2$,

$$\epsilon = \epsilon(0)[1-(gx)^2+(gy)^2] \tag{1}$$

$$g = \sqrt{(\epsilon_r-1)\Delta T/(\epsilon_r T_0)}/a, \quad \epsilon(0) = \epsilon_0\epsilon_r \tag{2}$$

where ϵ_0 and ϵ_r are the dielectric constant of the vacuum and specific dielectric constant of the gas medium at T_0, respectively. From (1), we can see that this medium has convergent properties in the transverse x direction and divergent properties in the y direction.

The characteristic spot size w_0 of this medium is given by the following equation [8],[1]

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[1] The characteristic spot size w_0 is a parameter which characterizes the convergency of a lens-like medium.

This corresponds to a parameter in Reference [4]. But in this paper, the definition of a spot size is of the Tien type [5], namely, the beam radius at which the beam intensity is down to $1/e$ of its maximum.

$$w_0/a = (1/\sqrt{k(0)g})/a = 1/[(k(0)a)^2(\epsilon_r - 1)\Delta T/T_0]^{1/2}, \tag{3}$$

where $k(0) = \omega\sqrt{\epsilon(0)\mu_0}$ is a propagation constant of a plane wave on the center axis. The parameters of the medium must be chosen so that $w_0/a < 1$ to utilize the convergent action of the medium.

When the axial length of the gas lens is finite, the edge effect must be taken into consideration. If the pipe separation $2a$ is much smaller than the axial length, such an effect may be neglected.

The heating power W of the medium per unit axial length is given by

$$W = 4\kappa\Delta T \quad \text{(watt/m)} \tag{4}$$

where κ is the specific heat conductivity of the medium. For example, in case of air for the gas medium, κ becomes 2.6×10^{-2} (watt/m °K), and so $W=0.13$ (watt/m), when $\Delta T = 2.5$°C.

Instead of using the hyperbolic cylinders as shown in Fig. 1, one may use circular cylinders as the heat conductors and obtain the nearly quadratic temperature distribution. To minimize the other nonquadratic terms in the distribution, we can choose the value of the radius r of the circular cylinders, given by the following relation [9],

$$r = 1.15a. \tag{5}$$

B. *Analysis of a Proposed Beam Waveguide*

The proposed beam waveguide is constructed with a number of hyperbolic-type gas lenses of axial length l_g as shown in Fig. 2. The spacing between the neighboring gas lenses is l_0, and adjacent gas lenses are rotated around the axis by 90° with respect to each other. Two neighboring gas lenses constitute a unit section, and the beam waveguide treated here is a periodic structure constructed by connecting many of the unit sections in tandem. According to Fig. 2, the gas lens at the left side has convergent properties, and the next has divergent properties, respectively, in the x direction.

In the y direction, on the contrary, the gas lens at the left side has divergent properties, and the next has convergent properties, so that the properties of the medium in the x or y directions are periodically repeated with the same periodicities but with mutual retardation of the half section in the x or y directions, respectively. Therefore, one may consider only one direction.

The natural mode of the waveguide is an axially asymmetric Hermite-Gaussian beam, and the spot size of the guided beam becomes maximum at the center of the convergent section ($=s_c$) and minimum at the center of the divergent section ($=s_d$) in the x or y direction.

Now, we specify the first half section of the unit section of the guide as from the center of the convergent component to the center of the next divergent component, as shown in Fig. 2. Then the $\tilde{F}$ matrix, which characterizes the wave property of a beam waveguide (Appendix), associated with the first half section is calculated as follows,

$$\tilde{F}_h = \begin{pmatrix} A_h & B_h \\ C_h & D_h \end{pmatrix}, \tag{6}$$

where

$$\left.\begin{aligned} A_h &= \cos\phi\cosh\phi - \sin\phi\sinh\phi - Q\sin\phi\cosh\phi \\ B_h &= jgk(0)[\sin\phi\cosh\phi - \sinh\phi\cos\phi \\ &\quad + Q\sinh\phi\sin\phi] \\ C_h &= j\frac{1}{gk(0)}[\sin\phi\cosh\phi + \sinh\phi\cos\phi \\ &\quad + Q\cos\phi\cosh\phi] \\ D_h &= \cos\phi\cosh\phi + \sin\phi\sinh\phi + Q\cos\phi\sinh\phi \end{aligned}\right\} \tag{7}$$

and,

$$w_0^2 = \frac{1}{gk(0)}, \quad \phi = gl_g/2, \quad Q = gl_0. \tag{8}$$

Here, ϕ is a normalized length of a gas-lens section and Q is a normalized spacing between neighboring gas lenses.

Using (41), (33), and (5), the normalized beam spot sizes s_c/w_0 and s_d/w_0 are calculated as follows.

$$\frac{s_c}{w_0} = \left[\cot^2\phi\cdot\frac{\tan\phi + \tanh\phi + Q}{-\cot\phi + \coth\phi + Q}\cdot\frac{\tan\phi + \coth\phi + Q}{\cot\phi - \tanh\phi - Q}\right]^{1/4}$$

$$\frac{s_d}{w_0} = \left[\coth^2\phi\cdot\frac{-\tan\phi + \tanh\phi + Q}{-\cot\phi + \coth\phi + Q}\cdot\frac{\cot\phi - \tanh\phi - Q}{\tan\phi + \coth\phi + Q}\right]^{1/4}. \tag{9}$$

In order to make use of the stable guide mode, the spot sizes s_c and s_d must be finite and positive real. This stability condition is obtained from (9) and expressed by the following equations.

$$Q + \tanh\phi < \cot\phi < Q + \coth\phi \tag{10}$$

$$-Q - \coth\phi < \tan\phi < -Q - \tanh\phi. \tag{11}$$

Equations (10) and (11) have the same meaning as the stability condition (40) obtained in the Appendix.

As shown in (10) and (11), there are many stable and unstable regions with respect to ϕ. In the stable region, the path of the center part of a Gaussian beam is also stable because the $\tilde{F}$ matrix serves also as a modified ray matrix which characterizes the ray property [13].

The relations between s_c/w_0 or s_d/w_0 and ϕ in the first stable region are shown in Fig. 3 with Q as a parameter.

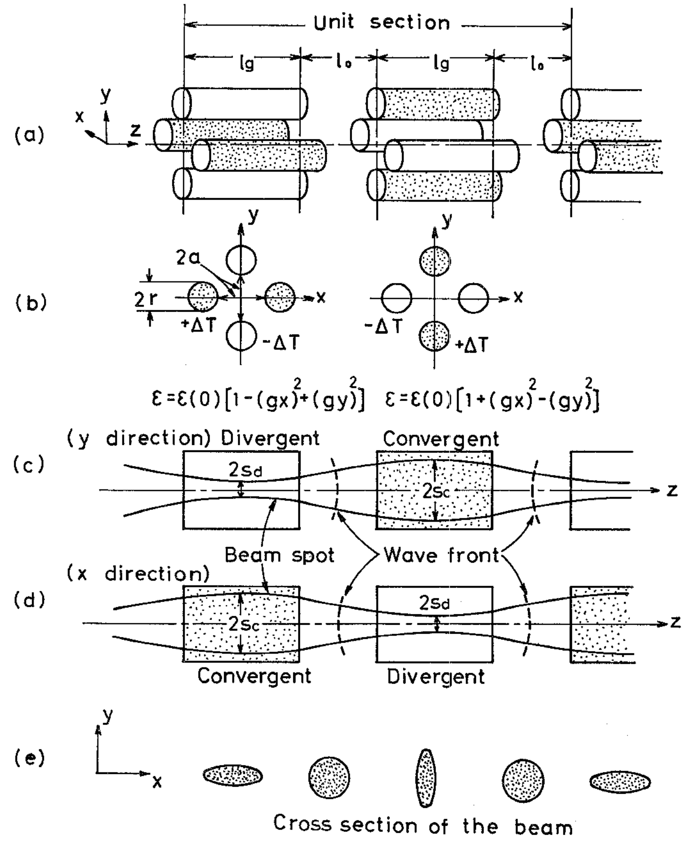

Fig. 2. (a) and (b) show a schematic view of beam waveguide using hyperbolic type gas lenses, each of which is periodically rotated by 90° around the center axis. (c) and (d) show the variation of the beam cross section in the y-z and x-z planes, respectively. (e) shows the cross section of the beam in the transverse plane.

If $\phi \gg 1$, and $Q \neq 0$, s_c/w_0 and s_d/w_0 are approximated as follows,

$$\left.\begin{aligned} \frac{s_c}{w_0} &\cong \frac{1}{\sqrt{\phi}}\left[\frac{1+\phi Q}{1-\phi Q}\right]^{1/4} \to \frac{1}{\sqrt{\phi}} \, (\phi \to 0) \\ \frac{s_d}{w_0} &\cong \frac{1}{\sqrt{\phi}}\left[\frac{1-\phi Q}{1+\phi Q}\right]^{1/4} \to \frac{1}{\sqrt{\phi}} \, (\phi \to 0) \end{aligned}\right\}. \quad (12)$$

C. Optimum Design of the Beam Waveguide

In the first stable region, s_c/w_0 has the minimum value $(s_c/w_0)_m$ at $\phi = \phi_m$ when Q is set to be constant. We can get the relation between ϕ_m and Q from Fig. 3 and it is shown in Fig. 4. In this figure, $(s_c/w_0)_m$ and $(s_c/w_0)_m$ at ϕ_m are also shown. When $\phi \gg 1$ and $Q \gg 1$ these relations can be approximated as follows,

$$\left.\begin{aligned} \phi_m &\cong 0.6180/Q \\ (s_c/w_0)_m &\cong 1.8248\sqrt{Q} = 1.4347/\sqrt{\phi_m} \end{aligned}\right\} \quad (13)$$

and these values are shown by the broken line in Fig. 4.

Let us define the optimum design of the beam waveguide, for which the maximum spot size s_c in the waveguide becomes minimum, which, in turn, minimizes diffraction losses and mode conversion effects as shown in the next section.

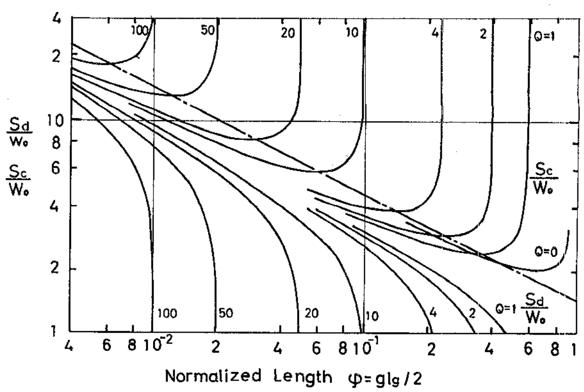

Fig. 3. Normalized spot sizes s_c/w_0 and s_d/w_0 vs. the normalized length $\phi = gl_g/2$ with a parameter as the normalized spacing $Q = gl_0$. s_c and s_d are spot sizes at the center of the convergent and the divergent component, respectively.

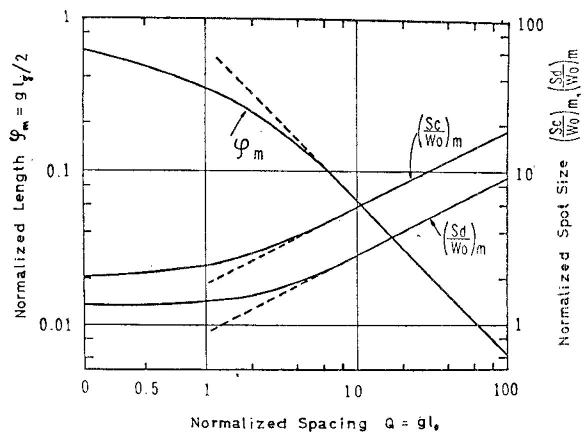

Fig. 4. ϕ_m, $(s_c/w_0)_m$, and $(s_d/w_0)_m$ vs. $Q = gl_0$ under the condition of (s_c/w_0) becomes minimum, namely, optimum condition.

At the optimum condition, the normalized length ϕ of the gas lens section becomes $\phi = \phi_m$.

Next, we consider the optimum design procedure of a waveguide using hyperbolic-type gas lenses. The separation $2a$ of the paired conductors must be set so that $s_c/a < 0.3$ to maintain small diffraction losses. This parameter is expressed by the following relation with the help of (3) as,

$$s_c/a = (s_c/w_0)_m(w_0/a)$$

$$= \left(\frac{s_c}{w_0}\right)_m \frac{1}{[k(0)^2(\epsilon_r - 1)\Delta T/(T_0\epsilon_r)]^{1/4}} \cdot \frac{1}{\sqrt{a}} \cdot \quad (14)$$

With this relation, we can determine the focusing parameter of the gas lens. When $(s_c/w_0)_m$ and w_0 are given, a is determined by (14).

At the optimum condition $\phi = \phi_m$, the length of the gas lens l_g and the spacing between gas lenses l_0 are expressed as follows, with the help of (8).

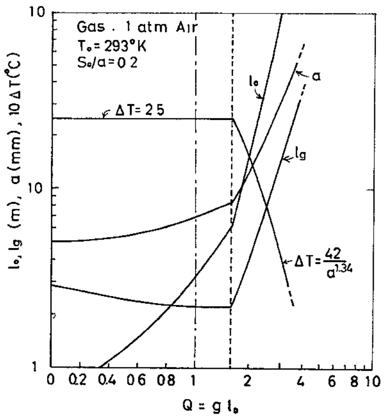

Fig. 5. The design chart which shows the relations between l_0, l_g, a, and $Q=gl_0$. Temperature difference ΔT is restricted, equal to 2.5°C, if $Q\leqq 1.6$ and $\Delta T=42/a^{1.34}$ if $Q>1.6$. The gas medium is ordinary air at room temperature and 1 atm.

TABLE I
NUMERICAL EXAMPLE OF THE PARAMETERS OF THE BEAM WAVEGUIDE

l_0 (m)	l_g (m)	a (mm)	s_c (mm)	ΔT (°C)	Q
0	2.7	5	1.0	2.5	0
10	2.9	10	2.0	2.0	1.8
50	8.8	22	4.4	0.71	2.6

$$l_g = \frac{2\phi_m}{g} = \frac{2\phi_m a}{[(\epsilon_r - 1)\Delta T/(\epsilon_r T_0)]^{1/2}} \tag{15}$$

$$l_0 = \frac{Q}{g} = \frac{Qa}{[(\epsilon_r - 1)\Delta T/(\epsilon_r T_0)]^{1/2}}. \tag{16}$$

Substituting the value of a obtained from (14), which corresponds to a given value of s_c/a, into (15) and (16), we can get the relations between Q and l_g or l_0.

In this way, if T_0, ΔT, $\epsilon_r - 1$, $k(0)$, s_c/a, and Q or l_0 are given, a and l_g are numerically determined.

It is confirmed experimentally, in Section V, that the maximum temperature difference ΔT (°C), when the operation of the gas lens is normal, is restricted by the value of spacing a (mm) as $\Delta T = 42/a^{1.34}$ for a gas medium of air with 1 atm at room temperature. Taking account of this fact, relations between Q and l_g, l_0, and a are, for example, shown in Fig. 5, where the gas medium is air with 1 atm at $T_0 = 293$°K, namely $\epsilon_r - 1 = 5.46\times 10^{-4}$, and $s_c/a = 0.2$, $k(0) = 9.929\times 10^6$ (m^{-1}), and $\lambda = 0.63$ μ. ΔT is chosen to be 2.5°C at a region of parameter Q being smaller than 1.6, and ΔT is chosen to be $42/a^{1.34}$ at a region of Q larger than 1.6. For this case, numerical examples are shown in Table I.

In case a slightly curved guide has a corresponding parameter of $l_0 = 0$ in Table I, and the beam center is displaced by Δx from the guide center axis to be guided smoothly along the guide, the guide radius of curvature R becomes $R = 4$ km, when $\Delta x = 1$ mm [6]. But if the guide is irregularly curved in a manufacturing process, the permissible guide radius of curvature should be much larger than 4 km [3], [4].

III. Diffraction Losses and Mode Conversion

A. Diffraction Losses

The effect of absorption of energy of the beam edge at the wall or at the heat conductive pipe of the gas lens can be regarded as diffraction. It is important to obtain the relation between the diffraction loss and the value of s_c/a.

Calculation of this relation seems to be difficult for this type of gas lens because the configuration of the wall is so complicated, but an approximation may be possible. According to Fig. 2, the effect of the wall on the absorption is most pronounced at the axial center of the convergent component because the beam spot becomes maximum at that point. When we consider this diffraction problem as a two-dimensional problem, it may be assumed that the effect of the wall of the gas lens is approximated by the effect of the black absorbing plate of infinite extent at the transverse direction with an aperture of width $2a$. This treatment is equivalent to the method of determining the diffraction loss in a Fabry-Perot resonator by Fox and Li [10].

We put the normalized electric field distribution in the x direction as

$$E(x) = \begin{cases} 1/(\sqrt{s_c}\,\sqrt[4]{\pi})e^{-1/2(x/s_c)^2} & |x| \leq a \\ = 0 & |x| > a \end{cases} \tag{17}$$

at the aperture. We extend $E(x)$ using the higher order Hermite-Gaussian function as follows:

$$E(x) = \frac{1}{\sqrt{s_c}\,\sqrt[4]{\pi}} \sum_{n=0}^{\infty} A_n H_n(x/s_c) e^{-1/2(x/s_c)^2}. \tag{18}$$

The amplitude A_0 of the fundamental mode can be obtained by using the orthonormality of the functions.

In this way, the diffraction loss per unit section of the guide, considering the effects both in x and y direction, is calculated as follows

$$\eta_d = 2(1 - |A_0|^2) = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{2X1}}^{\infty} e^{-1/2t^2} dt \tag{19}$$

where $X_1 = a/s_c$. The numerical relation of η_d vs. s_c/a is shown in Fig. 6.

B. Mode Conversion

If the distribution of dielectric constant at the transverse cross section in the gas lens contains nonquadratic terms, the transmitted Gaussian beam suffers mode conversion. In the case of hyperbolic-type gas lens, the distribution of the dielectric constant is more precisely calculated than (1) from Clausius-Mossotti's relation, and expressed as,

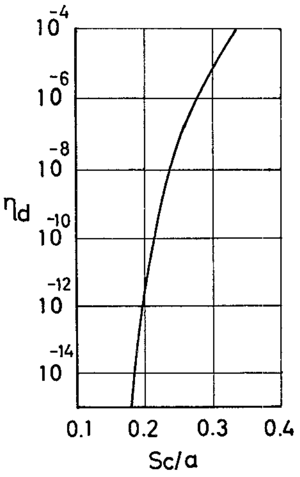

Fig. 6. Diffraction loss η_d per section vs. s_c/a.

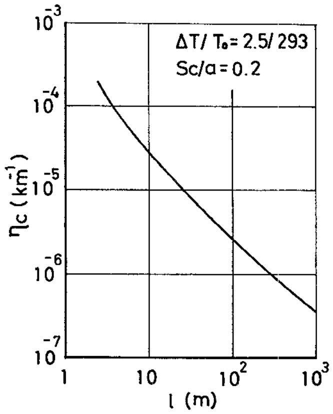

Fig. 7. The mode conversion ratio η_c per kilometer of proposed beam waveguide, in principle. $l=l_g+l_0$ is a distance between neighboring gas lenses.

$$\left.\begin{aligned}\epsilon &= \epsilon(0)\left[1-(gx)^2+(gy)^2+h^4(x^4+y^4+2x^2y^2)\right],\\ h^4/g^2 &= \Delta T/(T_0a^2),\end{aligned}\right\} \tag{20}$$

where only terms up to the fourth order are taken into account.

Because the fourth-order terms of x or y in (20) are usually small compared with the quadratic order terms, then the effect of the fourth-order terms can be regarded as small perturbations. In this way the mode conversion ratio η_c (=the power converted to higher modes/the power or the incident Gaussian beam) can be calculated with perturbation methods [7], and the η_c per unit section is given by the following equation:

$$\eta_c \cong (\Delta T/T_0)^2(\lambda lg/a^2)^2 D_1(t), \tag{21}$$

where

$$\left.\begin{aligned}t &= (s_c^2/w_0^2-1)^2/(s_c^2/w_0^2+1)^2\\ D_1(t) &= \left(\frac{3}{8\pi}\right)^2\frac{t(32t^2+56t+8)}{(1-t)^4}\end{aligned}\right\}. \tag{22}$$

The relation between η_c per kilometer and the length of the half section of the guide l $(=l_0+l_g)$ is shown in Fig. 7, at the optimum condition of the guide. The spot size ratio s_c/w_0 should be small in order to minimize the mode conversion effect.

The mode conversion due to the circular shape of the rod is neglected in the calculation of this perturbation problem with (20), if we choose the radius r of the rod as in (5), because the fourth-order terms of x or y in the representation of ϵ have disappeared [9].

IV. Mode Matching

If the beam entering this beam waveguide is an axially symmetric Gaussian mode, one has to match it to the natural mode of the guide [11], [12]. The mode matching is performed with, for example, two or three thin lenses as shown in Fig. 8.

In this case, we consider mode matching at the center part of the free space between two gas lenses. The waveform coefficients [8] of the natural mode at this point, namely $P_{\text{in},1}$ (x direction) and $P_{\text{in},2}$ (y direction), are calculated as follows, with the help of (39).

$$P_{\text{in},1} = P_{\text{in},2}^* = \frac{1}{w_{\text{in}}^2} + j\frac{k(0)}{f_{\text{in}}} \tag{23}$$

where $P_{\text{in},2}^*$ is a complex conjugate of $P_{\text{in},2}$ and,

$$\begin{aligned}w_{\text{in}} = w_0 &\left[\cos 2\phi\left\{Q\cosh 2\phi + \left(1+\frac{Q^2}{4}\right)\sinh 2\phi\right\}\right.\\ &+\left(\cosh 2\phi + \frac{Q}{2}\sinh 2\phi\right)\\ &\left.\cdot\left\{Q+\left(1+\frac{Q^2}{4}\right)\sin 2\phi\right\}\right]^{1/2}\\ &\times\left[1-\left\{\cos 2\phi\left(\cosh 2\phi+\frac{Q}{2}\sinh 2\phi\right)\right.\right.\\ &+\tfrac{1}{2}\sinh 2\phi\left(Q+\left(1+\frac{Q^2}{4}\right)\sin 2\phi\right)\\ &\left.\left.-\tfrac{1}{2}\sin 2\phi\left(Q\cosh 2\phi+\left(1+\frac{Q^2}{4}\right)\sinh 2\phi\right)\right\}^2\right]^{-1/4}\\ \frac{f_{\text{in}}}{k(0)} = -2w_0^2&\left[\cos 2\phi\left\{Q\cosh 2\phi+\left(1+\frac{Q^2}{4}\right)\sinh 2\phi\right\}\right.\\ &+\left(\cosh 2\phi+\frac{Q}{2}\sinh 2\phi\right)\\ &\left.\cdot\left\{Q+\left(1+\frac{Q^2}{4}\right)\sin 2\phi\right\}\right]\\ &\times\left[\sin 2\phi\left\{Q\cosh 2\phi+\left(1+\frac{Q^2}{4}\right)\sinh 2\phi\right\}\right.\\ &\left.+\sinh 2\phi\left\{Q+\left(1+\frac{Q^2}{4}\right)\sin 2\phi\right\}\right]^{-1}.\end{aligned} \tag{24}$$

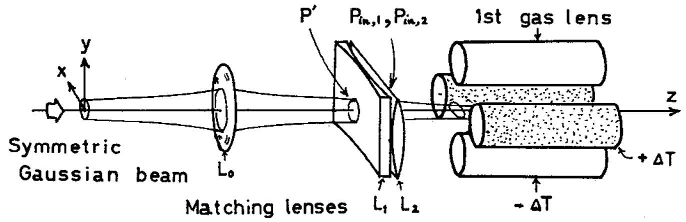

Fig. 8. A method of mode matching.

According to Fig. 8, the first symmetric lens L_0 is placed at a point on the center axis in order to match the incident spot size to become w_{in} at the matching point. If the waveform coefficient at the matching point is P' given by

$$P' = \frac{1}{w_{\text{in}}^2} + j\frac{k(0)}{f'} \qquad (x \text{ and } y \text{ direction}), \tag{25}$$

then we must adjust the imaginary part of P' to match the imaginary parts of $P_{\text{in},1}$ and $P_{\text{in},2}$ in x and y directions. For this purpose, two-dimensional thin convex and concave lenses L_1 and L_2, whose focal lengths are f_1 and f_2, respectively, are useful. The values of f_1 and f_2, for which the matching condition is obtained, are given by the following equation, using (23) and (25).

$$\left.\begin{aligned} f_1 &= -\frac{f' f_{\text{in}}}{f' - f_{\text{in}}} \\ f_2 &= \frac{f' f_{\text{in}}}{f' + f_{\text{in}}} \end{aligned}\right\}. \tag{26}$$

V. Experiments with a Hyperbolic-Type Gas Lens

To confirm the design theory of the hyperbolic-type gas lens, we made some experiments with a hyperbolic-type gas lens. Four types of the gas lenses, type A, B, C, and D, with different dimensions were used. The cross-sectional view of the experimental gas lenses of type A, B, and D are shown in Fig. 9(a), (b). Circular cylinders made of brass with radius r were used instead of ideal pipes with hyperbolic cross sections. The gas medium and some parts of the rods are insulated by a cardboard from the surroundings, as shown in Fig. 9 (a), (b) by a dotted line. At type C, cardboards are inserted between the hot and cooled brass cylinders acting as thermal insulators, as shown in Fig. 1. The dimensions of the experimental gas lenses are shown in Table II.

The temperature of the cooled pair of pipes was held at room temperature with radiators and the hot pair were heated by electric heaters. The gas medium was air of 1 atm and room temperature. Thermocouple probes were used for measuring the wall temperature. The temperature of the paired pipes was made to be identical and uniform along the axial direction.

In the case of type A and type B gas lenses, TEM_{00} Gaussian beam modes with wavelength $\lambda = 0.63\ \mu$ were used to measure the convergency. The output beam leaving the experimental gas lens was deformed to an axially asymmetric pattern as in Fig. 9(d) or (e). Let us denote the spot sizes in the x and y directions as w_x and w_y, respectively.

At first, the waveform coefficient of the laser beam at the input boundary of the experimental gas lens, $P_{\text{in}} = 1/w_{\text{in}}^2 + jk/R_{\text{in}}$ was determined theoretically using the analysis of the Fabry-Perot resonator mode of He–Ne gas laser used here. The measured imaginary part of P_{in} was in good agreement with the theoretical value of P_{in}. Next, the theoretical waveform coefficient at the output boundary of the gas lens was calculated using (34). Let us denote the waveform coefficient as $P_{ox} = U_{ox} + jV_{ox}$ in the x direction and $P_{oy} = U_{oy} + jV_{oy}$ in the y direction, respectively.

When the beam from the gas lens travels a distance L in free space, the spot sizes w_x and w_y at this point are given by (34), and the ratio w_x/w_y is given by the following equation,

$$\frac{w_x}{w_y} = \sqrt{\frac{U_{ox}}{U_{oy}} \cdot \frac{U_{oy}^2 + (V_{oy}^2 + k/L)^2}{U_{ox}^2 + (V_{ox}^2 + k/L)^2}}. \tag{27}$$

The measured ratio w_x/w_y or w_y/w_x vs. the distance from the gas lens L is shown in Fig. 10 with corresponding theoretical values. For these lenses with $2a = 10$ mm, the effects of convection and gravity were not experienced as long as the temperature difference ΔT was lower than approximately 5°C.

In the case of the type C and type D lenses, an autocollimator was used to measure its convergency, as shown in Fig. 11(a).

The image of a cross from an autocollimator was incident upon the experimental hyperbolic-type gas lens whose two transverse axes coincided with the two directions along the image of a cross, and reflected back by a flat mirror with the surface irregularity of smaller than $\lambda/10$.

The reflected image of the ray lost its sharpness because of the convex or concave lens action in x direction or y direction, respectively. Seeing the gas lens from the autocollimater side, we can approximately consider the gas lens as an equivalent mirror with curvature of $1/R(<0)$ in the x direction and $1/R(>0)$ in the y direction, respectively. This lens effect in the x direction or y direction could be compensated by displacing the objective O, and the curvature could be measured from the displacement.

In case of type C gas lens, as an example, the measured curvature is shown in Fig. 11 vs. ΔT. Experimental errors are several percents. The lens action becomes abnormal when ΔT was over approximately 0.9°C. This is caused by the effect of convection due to gravity.

In case of the type D gas lens with $2a = 19$ mm, the lens action became abnormal when ΔT was over approximately 2.2°C.

When $g \cdot l_g$ is very small, the equivalent curvature $1/R$ is approximated by the following equation

TABLE II
DIMENSION OF THE EXPERIMENTAL GAS LENSES

Type	Pipe Separation $2a$ (mm)	Diameter of Brass Cylinder $2r$ (mm)	Length l_g(mm)	Pipe Construction	Connection	Maximum Temperature Difference ΔT_m (°C)
A	10	10	1.0	Fig. 9(a)	single	5.0
B	10	10	4.2	Fig. 9(b)	two Fig. 9(e)	5.0
C	38	51	2.1	Fig. 9(e) alike	single	0.85
D	19	19	0.5	Fig. 9(e)	single	2.2

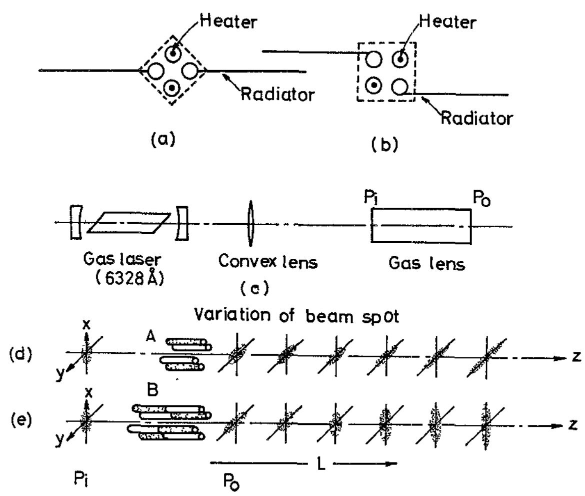

Fig. 9. Experimental arrangement on the measurement of the convergent property of hyperbolic-type gas lens.

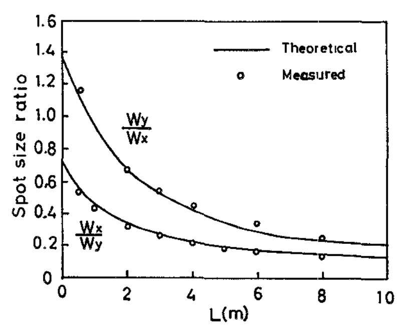

Fig. 10. Experimental results on spot size ratio w_x/w_y or w_y/w_x vs. L which is the distance between the output end of the gas lens and the measured position. In case of (A), the measured values of w_x/w_y correspond to a response of a single-section type-A gas lens as shown in Fig. 9 with $2\Delta T=5.5$°C. In case of (B), values of w_y/w_x correspond to a response of a two-section type-B gas lens as shown in Fig. 4.(e) with $2\Delta T=11.7$°C.

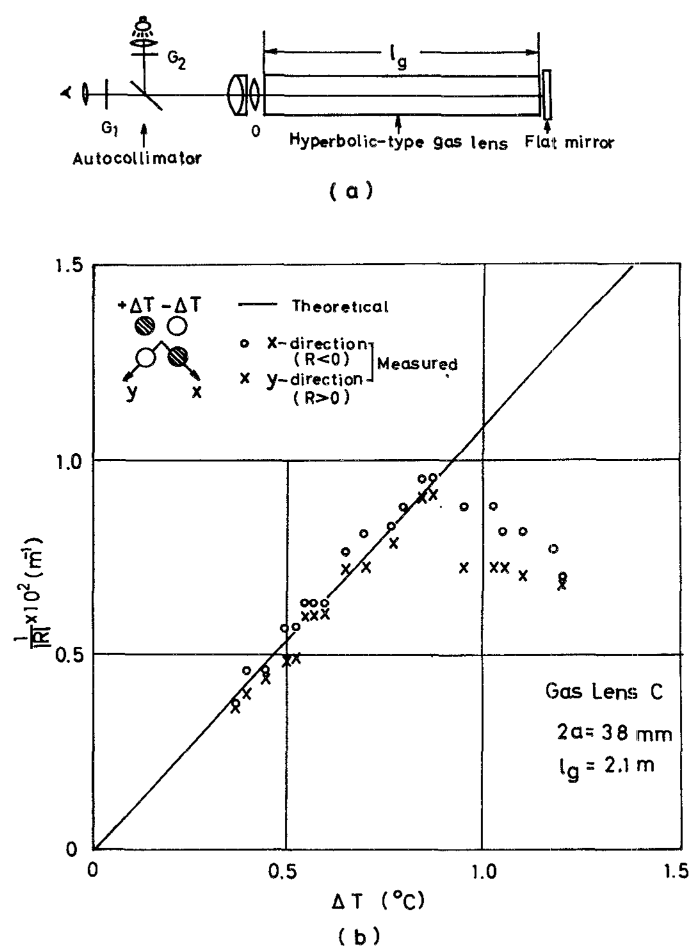

Fig. 11. (a) Experimental arrangement on measurement of the equivalent curvature with an autocollimator. (b) Experimental and theoretical results of the equivalent curvature of the gas lens whose gas medium is ordinary air at room temperature.

$$\frac{1}{|R|} \cong g^2 l_g = \frac{(\epsilon_r - 1)\Delta T}{\epsilon_r T_0} \cdot \frac{l_g}{a} \cdot \qquad (28)$$

From (28) we can see that $1/R$ is proportional to ΔT and is shown in Fig. 11 as the theoretical value with parameters used in the experiments.

Measured results were in agreement with the theoretical values within the experimental errors.

For the type C gas lens, the temperature distribution in the gas lens medium has been measured by a thermister probe. It was quadratic as was expected from the theory. The construction shown in Fig. 9(b) was more preferable than the configuration shown in Fig. 9(a) to realize the symmetric temperature distribution when ΔT became close to the maximum value.

The maximum temperature difference ΔT_m, at which the normal operation of the hyperbolic-type gas lens is confirmed, is restricted by the pipe separation $2a$. It is shown in Table II. The relation between the maximum ΔT_m(°C) and a(mm) is approximated experimentally as follows,

$$\Delta T_m \times a^{1.34} \leqq 42 \tag{29}$$

for the ordinary air at $T_0=293°K$ and at the region of between approximately 10 mm and 40 mm.

VI. Conclusion

In conclusion, this type of light beam waveguide has the following merits. The temperature distribution is ideally quadratic in the transverse directions, so that the mode conversion due to the higher order variation of the dielectric constant can be minimized. Moreover, the design procedure is simple, and the mode matching at the input boundary can be accomplished easily. Also it is possible to use ordinary air as the lens medium. The theoretical treatment given here can easily be extended to the curved guide [6], [8].

Three kinds of experimental hyperbolic-type gas lenses were made and the operating temperature difference $2\Delta T$, in which the normal operation of the gas lens is confirmed without any effects of the convection and the gravity, was determined experimentally.

Appendix

Theoretical Basis [8]

The beam mode of the transverse field distribution in a lens-like medium whose dielectric constant is given by (1) is expressed by a Hermite–Gaussian function [4], [8] and given by the following equation,

$$E = E_0\sqrt{\frac{s_1s_2}{w_1w_2}}H_p\left(\frac{x}{w_1}\right)H_q\left(\frac{y}{w_2}\right) \cdot\exp\left[-\frac{1}{2}\left(\frac{1}{w_1^2}+j\frac{k(0)}{R_1}\right)x^2 -\frac{1}{2}\left(\frac{1}{w_2^2}+j\frac{k(0)}{R_2}\right)y^2\right] \times\exp\left[-jk(0)z+j\left(p+\frac{1}{2}\right)\tan^{-1}F_1 +j\left(q+\frac{1}{2}\right)\tan^{-1}F_2\right], \tag{30}$$

where s_1 and s_2 are the beam spot size at $z=0$ with a condition of planary equiphase surface. The spot size w_1, w_2 and the radius of the equiphase surface R_1, R_2 at axial distance z are given by the following relation.

$$\left.\begin{aligned} w_1 &= s_1\sqrt{\cos^2 gz+\left(\frac{w_0}{s_1}\right)^2\sin^2 gz}\\ R_1 &= 2k(0)w_0^2\frac{\cos^2 gz+\left(\frac{w_0}{s_1}\right)^4\sin^2 gz}{\left[\left(\frac{w_0}{s_1}\right)^4-1\right]\sin 2gz}\\ &\text{with}\\ F_1 &= \left(\frac{w_0}{s_1}\right)^2\tan gz \end{aligned}\right\} \tag{31}$$

and

$$\left.\begin{aligned} w_2 &= s_2\sqrt{\cosh^2 gz+\left(\frac{w_0}{s_2}\right)^4\sinh^2 gz}\\ R_2 &= 2k(0)w_0^2\frac{\cosh^2 gz+\left(\frac{w_0}{s_1}\right)^4\sinh^2 gz}{\left[\left(\frac{w_0}{s_1}\right)^4+1\right]\sinh 2gz}\\ &\text{with}\\ F_2 &= \left(\frac{w_0}{s_2}\right)^2\tanh gz. \end{aligned}\right\} \tag{32}$$

We define a waveform coefficient P_m as

$$P_m = \frac{1}{w_m^2}+j\frac{k(0)}{R_m} \qquad (m=1,2) \tag{33}$$

which is related to the coefficient of x^2 or y^2 in the exponent of (30). In case that a Gaussian beam whose waveform coefficient is P_{im} is transmitted through a beam waveguide by a length z, and the waveform coefficient at the point is transformed to be P_{om}, then P_{im} is related to P_{om} by the following equation.

$$P_{im} = \frac{A_mP_{om}+B_m}{C_mP_{om}+D_m} \qquad (m=1,2). \tag{34}$$

Here A_m, B_m, C_m, and D_m are components of the $\tilde{F}$ matrix of the beam waveguide in each direction x and y, respectively [8]. $\tilde{F}$ matrices associated with a convergent and divergent lens-like medium, and a free space with length z is expressed as follows:

convergent medium
$$\begin{bmatrix} \cos gz & jk(0)g\cdot\sin gz\\ j\frac{1}{k(0)g}\sin gz & \cos gz \end{bmatrix} \tag{35}$$

divergent medium
$$\begin{bmatrix} \cosh gz & -jk(0)g\cdot\sinh gz\\ j\frac{1}{k(0)g}\sinh gz & \cosh gz \end{bmatrix} \tag{36}$$

free space
$$\begin{bmatrix} 1 & 0\\ j\frac{z}{k} & 1 \end{bmatrix}. \tag{37}$$

When the different lens-like media, whose $\tilde{F}$ matrices are $\tilde{F}_1, \tilde{F}_2, \cdots, \tilde{F}_n$, are connected in tandem, the total $\tilde{F}$ matrix is given by the product of them

$$\tilde{F} = \tilde{F}_1\cdot\tilde{F}_2,\cdots,\tilde{F}_n. \tag{38}$$

If a beam waveguide is constructed with a tandem connection of unit sections, each of which has elements of $\tilde{F}$ matrix, namely, A_m, $B_m=jB'_m$, $C_m=jC'_m$, and D_m, the waveform coefficient P_m of the natural mode at the input boundary of each unit section is given as follows,

$$P_m=\left[\pm\sqrt{4-(A_m+D_m)^2}-j(A_m-D_m)\right]/(2C_m') \tag{39}$$

where the sign which must be set as the real part of P_m becomes positive. The stability condition is given as follows, using (39)

$$|A_m + D_m| \leqq 2. \tag{40}$$

When the unit section has a symmetric property in the axial direction and the elements of the $\tilde{F}$ matrix of the first half section are A_h, B_h, C_h, and D_h, then

$$P_m = \sqrt{A_h B_h / C_h D_h} \tag{41}$$

and the stability condition is as follows,

$$A_h B_h C_h D_h \leqq 0. \tag{42}$$

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A Switching Circulator: S-Band; Stripline; Remanent; 15 kilowatts; 10 microseconds; Temperature-Stable

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***Abstract*—A stripline, three-port remanence circulator switch has been designed for high-speed switching of time delay in a phased array radar at S-band (2.9 GHz). Special attention was devoted to minimizing switching time and energy through design of the magnetic circuit and suppression of eddy currents. Temperature stabilization of insertion phase was accomplished by means of a flux regulating magnetic circuit. Switching performance: time: less than 10 microseconds; energy: 450 microjoules. Circulator performance: bandwidth for >26 dB isolation, 8.9 percent; insertion loss, 0.35 dB. Temperature stability of insertion phase: one electrical degree per 10°C. Peak RF power: 15 kW. The discussion includes details of the junction design and performance, techniques of eddy current suppression, temperature stabilization, and the method of switching energy measurements.**

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Introduction

THIS PAPER reports the development of a ferrite switching circulator suitable for time delay switching applications in phased array radars. The re-